# Heat Transfer Enhancement in a Parabolic Trough Solar Receiver using Longitudinal Fins and Nanofluids

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In this paper, we present a three dimensional numerical investigation of heat transfer in a parabolic trough collector receiver with longitudinal fins using different kinds of nanofluid, with an operational temperature of 573 K and nanoparticle concentration of 1% in volume. The outer surface of the absorber receives a non-uniform heat flux, which is obtained by using the Monte Carlo ray tracing technique. The numerical results are contrasted with empirical results available in the open literature. A significant improvement of heat transfer is derived when the Reynolds number varies in the range 2.57×104 ≤ Re ≤ 2.57×105, the tube-side Nusselt number increases from 1.3 to 1.8 times, also the metallic nanoparticles improve heat transfer greatly than other nanoparticles, combining both mechanisms provides better heat transfer and higher thermo-hydraulic performance.

**Keywords: numerical study, Monte Carlo ray trace, parabolic trough collector, heat transfer, longitudinal fins, nanofluid**

## Introduction

THE surge in fossil fuels prices during the petrol crisis launched the industrialized countries in the race for alternative and renewable energies such as solar energy. This source of energy is considered as the most economical and clean. At the same time, solar thermal power plants have been the subject of study among the technology of parabolic trough collectors, which are currently the most proven SOLAR CONCENTRATION TECHNIQUES [1]. Several studies have recently focused on the tube-side heat transfer enhancement of these devices, following both numerical and experimental methodologies. The descriptions of the convective heat transfer, the effect of the geometry and the use of different working fluids have been analyzed by these authors.

Aggrey et al. [2] presented a numerical investigation of thermal performance of receiver for a parabolic trough collector (PTC) with perforated plate inserts. Their results show that the use of inserts improve the thermodynamic performance of the receiver by minimizing the entropy generation rates, and described the dependence of the Nusselt number and friction factor on the spacing and size of the insert. Wang et al. [3] investigated numerically the heat transfer enhancement in the receiver tube of a direct steam generation system with parabolic trough by inserting metal foams; they reported the significant effect of the layout and dimensionless height of metal foams on the thermal performance greatly, whereas the porosity of the foam proved to have a slight influence on the heat transfer. Song et al. [4] analyzed the heat transfer enhancement of PTC receiver with non-uniform heat flux and helical screw-tape inserts; their results indicate that the maximum temperature on the outer surface

**Nomenclature**

| | | | |
|---|---|---|---|
| A | area, $m^2$ | $\rho$ | Density, $kg/m^3$ |
| $C_p$ | Specific heat, J/ kg K | $\lambda$ | thermal conductivity, W/m K |
| d | Diameter, m | $\mu$ | Viscosity, Pa s |

| | | | |
|---|---|---|---|
| DNI | Direct normal irradiance, $W/m^2$ | $\eta$ | thermo-hydraulic performance |
| f | Friction factor | **Subscripts** | |
| h | Heat transfer coefficient, $W/m^2K$ | amb | Ambient |
| $h_W$ | Glass cover outer heat transfer coefficient, $W/m^2K$ | b | Bulk fluid state |
| L | Receiver length, m | bf | Base fluid |
| $\dot{m}$ | Mass flow rate, kg/s | f | Fluid |
| Nu | Nusselt number | go | Outer glass cover wall |
| P | Pressure, Pa | in | Inlet |
| PEC | Performance evaluation criteria | nf | Nanofluid |
| Pr | Prandtl number | p | particle |
| $\overline{q''}$ | Heat flux, $W/m^2$ | ri | absorber tube inner wall |
| Re | Reynolds number | sky | sky temperature |
| T | Temperature, K | w | wall |
| $v_w$ | Wind velocity, m/s | **Abbreviation** | |
| **Greek letters** | | HTF | heat transfer fluid |
| $\varphi$ | particle volume concentration | PTC | parabolic trough collector |
| $\varepsilon$ | emissivity | | |

of the absorber tube increases along with inlet temperature and solar irradiation. Cheng et al. [5] carried out a numerical study of heat transfer enhancement by unilateral longitudinal vortex generators inside PTC receiver. They illustrated that the average Nusselt number and average friction factor increase with increasing each geometric parameter, whereas the thermal loss decreases with the increase of each geometric parameter.

Recently, a new class of fluids called nanofluid has been developed and tested, this term was proposed by Choi in 1995 [6] at Argonne National Laboratory; as a liquid mixture with a small concentration of nanometer-sized solid particles in suspension. Nanofluids have interesting thermo-physical properties such as high thermal conductivity.

The researches on the application of nanofluids have been popularized during the recent years; various authors have investigated the effects of using nanofluid on heat transfer enhancement inside PTC receiver. Sokhansefat et al. [7] studied the effect of using $Al_2O_3$/synthetic oil in a PTC tube, reporting that heat transfer augments for increasing nanoparticle volume fraction and operational temperature. Risi et al. [8] investigated the heat transfer enhancement for CuO+Ni/nitrogen gas in a PTC tube, demonstrating that above 0.3 %vol the drawback effect of pressure drop overwhelm the beneficial effects of thermal properties, additionally the optimization procedure found a maximum solar to thermal efficiency equal to 62.5%.

The present work prospects the use of a compound enhancement technique for parabolic trough collector, based on the use of nanofluids and the presence of two longitudinal fins in the tube side of the PTC. A three dimensional numerical model is implemented in ANSYS Fluent for the solution of the flow field and heat transfer in the enhanced geometry. The heat flux around the absorber tube was obtained applying MCRT (Monte Carlo ray tracing) method. The first part of this study analyzes the effect of using longitudinal fins inserts when the Reynolds number varies in the range $2.57\times10^4 \le Re \le 2.57\times10^5$ depending on the heat transfer fluid characteristics. In the last part we investigate a comparison between four different kinds of nanoparticles, with nanoparticle concentration of 1 % in volume. The aim of this paper is to develop the influence of heat transfer fluid properties and receiver geometries of a parabolic trough solar collector.

## Physical model

In our investigation, we considered a simple model of receiver of the parabolic trough solar collector, in which all effects of the central rod and other supports are considered negligible.

For our analysis, the solar collector which is shown in Fig. 1 is chosen as the geometrical model in this simulation. The materials used for the glass cover and the absorber tube are respectively the borosilicate glass and steel. The space between both tubes is considered as a vacuum at low pressure and ambient temperature.

The physical parameters used in this study are given in Table1.

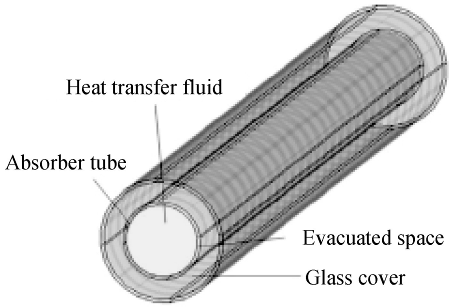



**Fig. 1** Schematic of PTC receiver

**Table 1** Receiver dimension.

| | |
|---|---|
| Focal length | 1.71 m |
| Aperture width | 5.77 m |
| Absorber inner radius | 3.2 cm |
| Absorber outer radius | 3.5 cm |
| Glass cover inner radius | 5.96 cm |
| Glass cover outer radius | 6.25 cm |
| Material of the absorber | Steel |
| Material of the glass envelope | Borosilicate |
| Transmittance of glass cover | >96% |
| Coating absorbance | 95% |
| Glass cover emissivity | 0.837 |

The objective of our study is to improve the heat transfer inside parabolic trough collector receiver, for which we analyze the effect of longitudinal fins inserted in the absorber tube. Fig. 2 shows the fin's physical model utilized in this work.

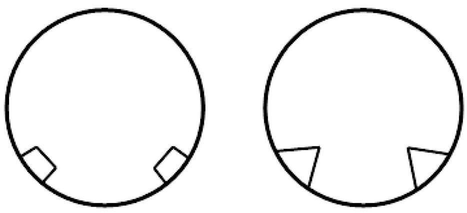



**Fig. 2** Physical model of absorber with longitudinal fins inserts.

## Numerical model

The numerical solution is formed and meshed by using the commercial software GAMBIT 2.4.6; it's also utilized for setting and specifying the boundary conditions. The turbulent model used in this study is the k-ω SST [9]. The governing equations such as the continuity, momentum, energy and other scalars are solved by using the Finite volume solver Fluent 6.3.26 [10].

The finite volume technique converts the non-linear partial differential equations with the first order upwind scheme. The pressure-based solver is used to solve the pressure based equation.

### Heat transfer fluid properties:

In the first part of this study, the heat transfer fluid (HTF) used is synthetic oil DOWTHERM A. this is a eutectic mixture of 73% Diphenyl Oxide ($C_2H_{10}O$) and 27% Biphenyl ($C_2H_{10}$). This fluid exhibits favorable physical properties and low vapor pressure at the maximum operating temperature [11].

The thermo-physical properties of DOWTHERM A as a function of temperature are provided in Appendix A. Table 2 shows the physical characteristics of base fluid at inlet temperature 573 K.

**Table 2** Thermo-physical properties of DOWTHERM A.

| | |
|---|---|
| Density (kg/m$^3$) | 803.3 |
| Specific heat (J/kg K) | 2373 |
| Thermal conductivity(W/mK) | 0.093 |
| Viscosity (mPa s) | 0.2 |

### Numerical modeling

For this study, the outer wall of the absorber tube receives a non-uniform heat flux; this distribution was obtained by using the Monte Carlo ray trace technique [12]. Fig. 3 illustrates the simulation results of the local concentration ratio (LCR) distribution on a cross-section of the absorber outer surface. The LCR is defined as the ratio of the concentrated radiant flux at a local position on the receiver surface to the direct normal irradiance (DNI), where a DNI of 1000 W/m$^2$ was used in this work. Symmetry boundary condition is utilized for the inlet and the outlet of the space between the absorber and the glass cover. For the outer glass cover, a thermal boundary condition that combines the convection and radiation heat transfer is used. Glass emissivity is about 0.83 and sky emissivity is determined by using the correlation proposed by Martin and Berdahl [13] given by:

$$\varepsilon_{sky} = 0.711 + 0.56\frac{T_{dp} - 273.15}{100} + 0.73\left(\frac{T_{dp} - 273.15}{100}\right)^2 \quad (1)$$

Sky temperature can be calculated using the following correlation [14]:

$$T_{sky} = 0.0552\, T_{amb}^{1.5} \quad (2)$$

where the ambient temperature used in this simulation is 300 K and $T_{dp}$ is dew point temperature (K).

Additionally, the convection heat transfer coefficient

used for the boundary condition is defined by the experimental correlation [15]:

$$h_w = 4v_w^{0.58} d_{go}^{-0.42} \tag{3}$$

where: $v_w$ is the wind speed (2 m/s in this study) and $d_{go}$ is the glass envelope outer diameter.

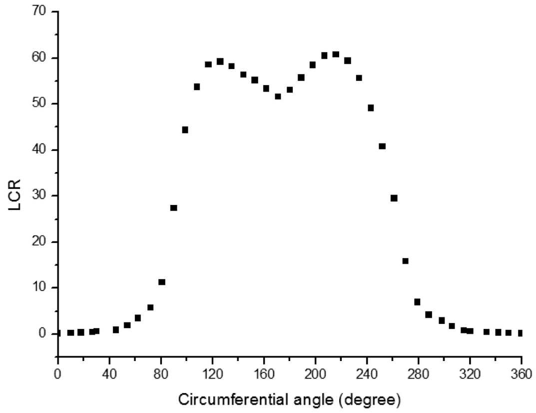



**Fig. 3** The local concentration ratio on a cross-section of the absorber outer surface

## Results and discussion

### Validation of numerical results

The average Nusselt number and heat transfer coefficient are defined as:

$$\overline{Nu} = \frac{h\, d_{ri}}{\lambda} \tag{4}$$

$$\overline{h} = \frac{\overline{q''}}{T_w - T_b} \tag{5}$$

Where $\overline{q''}$ is the average heat flux on absorber tube's inner wall; $T_w$ is the average inner wall temperature of the absorber tube and $T_b$ is the average bulk temperature of the HTF.

Also the Darcy friction factor for turbulent flow is defined in the following relation:

$$f = \frac{\Delta p}{\frac{L}{d_{ri}} \rho \frac{v^2}{2}} \tag{6}$$

For validate purposes, the numerical results of Darcy friction factor are compared with the correlations proposed by Petukhov [16] and Blasius [17]; likewise the numerical results of average Nusselt number are compared with the correlations suggested by Gnielinski [18] and Notter-Rouse [19] which are given as follows:

Gnielinski correlation:

$$Nu = \frac{\frac{f}{8}(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7\left(\frac{f}{8}\right)^{0.5}\left(\mathrm{Pr}^{\frac{2}{3}}-1\right)} \tag{7}$$

where Re and Pr are respectively the Reynolds number and Prandtl number.

In addition, $f$ is the friction factor calculated by Petukhov's correlation defined as:

$$f = (0.79\ln\mathrm{Re} - 1.64)^{-2} \tag{8}$$

where: $10^4 \le \mathrm{Re} \le 5\times10^6$

and $0.5 \le \mathrm{Pr} \le 2000$

Notter-Rouse represents the correlation of average Nusselt number as follows:

$$Nu = 5 + 0.015\mathrm{Re}^{0.856}\,\mathrm{Pr}^{0.347} \tag{9}$$

Blasius proposed a correlation for the calculation of friction factor which is given by:

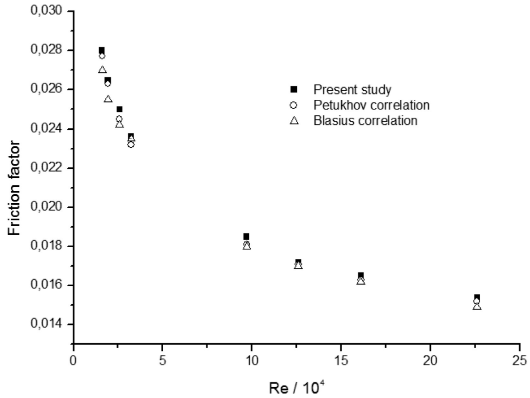
$$f = 0.316\mathrm{Re}^{-0.25} \tag{10}$$

where $\mathrm{Re} \le 2\times10^4$

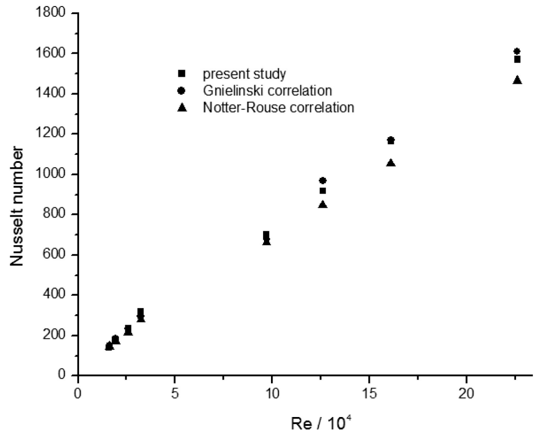
$$f = 0.184\mathrm{Re}^{-0.2} \tag{11}$$

where $\mathrm{Re} \ge 2\times10^4$

Fig.4 (a) shows compatible results of the Darcy friction factor between the numerical results and the empirical correlations. The maximum deviation is around 4.1% and the minimum relative error is about 0.7%. Also; Fig. 4 (b) represents the results of average Nusselt number, the maximum and the minimum deviation between our numerical results and the correlation of Gnielinski are 4.8% and 0.64% respectively. Additionally; the maximum and




(a) The results of Darcy friction factor




(b) The results of Nusselt number

**Fig. 4** Validation of numerical results for smooth absorber.

minimum error between our results and the correlation of Notter-Rouse are 9.3% and 1.8%. These results demonstrate that there is a good agreement between the present numerical results and those obtained by the empirical correlations.

**The effect of using fins inserts in the absorber tube**

**Thermal performance analysis:**

The solution of the absorber with insert fins retrieves a higher Nusselt number, compared with the smooth tube model. Fig. 5 presents the Nusselt number evolution with Reynolds number for smooth tube absorber and for the case-1 and case-2 geometries three. Nusselt number augmentations between 1.3 to 1.8 times compared to the plain tube are reported, which means that heat transfer is enhanced greatly by inserting longitudinal fins.

Fig. 6 shows that the Darcy friction factor in tube with fins inserts is higher than the empty tube, and it decreases with increasing Reynolds number. This higher friction factor is the results of the swirling flow induced by the longitudinal inserts that act like an obstacle.

In order to evaluate the heat transfer enhancement, we use the thermal performance criteria defined as the ratio of the dimensionless Nusselt number and the dimensionless friction factor, given by the following relation [20]:

$$PEC = \frac{\frac{Nu}{Nu_0}}{\left(\frac{f}{f_0}\right)^{\frac{1}{3}}} \qquad (12)$$

where the subscript 0 refers to the solution of the smooth tube.

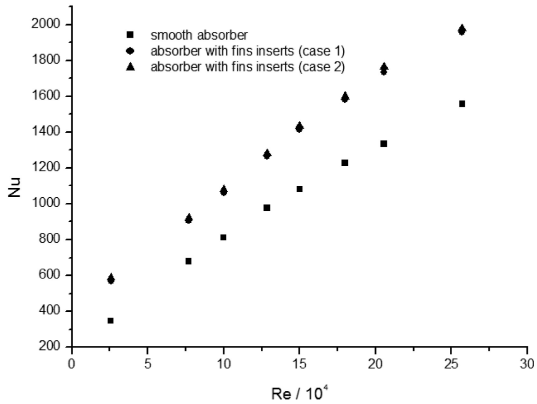



**Fig. 5** Variation of Nusselt number with Reynolds number in absorber with and without fins.

As expected, the thermal performance decreases with increasing Re, as is clearly shown in Fig. 7. An average value of PEC≈1.5 is reported in the range $2.57\times10^4 \leq Re \leq 2.57\times10^5$. The thermal performance of the absorber with triangular fins is slightly greater than that of tube with rectangular fins which means that the geometric parameters of fins have a noticeable effect in heat transfer enhancement. The effects of the inserts on thermal performance factor are also principally governed by the influence of the heat transfer improvement.

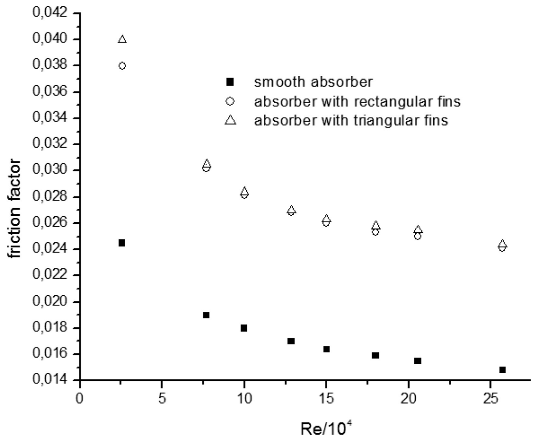



**Fig. 6** Variation of friction factor with Reynolds number in tube with and without fins.

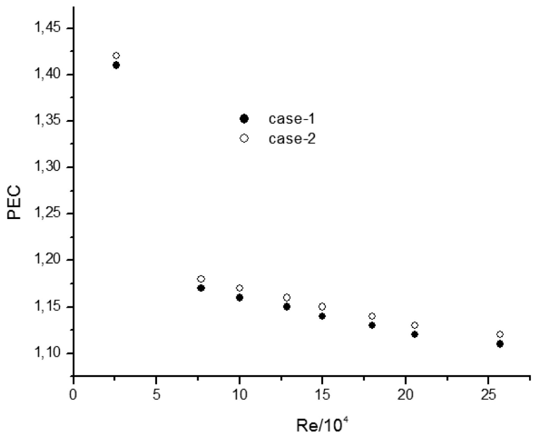



**Fig. 7** Evaluation of thermal performance for absorber with fins inserts.

**Circumferential temperature analysis**

Fig. 8 presents the temperature distribution on the middle cross-section of the absorber inner surface with and without fins when the DNI, the HTF inlet velocity and the HTF inlet temperature are 1000 W/m$^2$, 1 m/s and 573 K, respectively. It can be observed that the temperature of the absorber inner wall for tube with fins is higher than the smooth tube, especially where the fins are inserted.

Also the HTF temperature is augmented by inserting the longitudinal fins when the difference is about 13 K; Fig. 9 illustrates that the temperature of the fluid on the bottom is higher than that on the top; this is due to the non-uniform heat flux distribution which is higher in the bottom.

**The effects of using nanofluids as HTF**

In recent years, a lot of researchers have investigated the effects of nanofluids on the enhancement of heat transfer in thermal engineering.

According to a recent research, adding nanoparticles into a base fluid can improve the thermo-physical properties of the heat transfer fluid. The nanoparticle volume fraction has a remarkable effect on heat transfer.

In this section we compare between four kinds of nanoparticles on the performance of a parabolic trough solar collector. The nanoparticles used are:

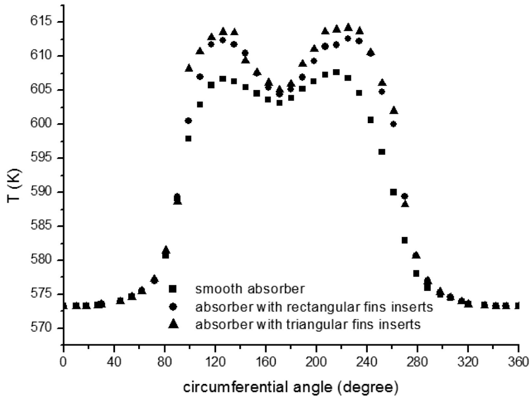



**Fig. 8** Temperature distribution on the middle cross-section of the absorber inner wall.

- Oxide ceramic ($Al_2O_3$),
- Metal (Cu),
- Metal carbide (Sic),
- Nonmetal (C).

The correlations used for calculating the thermo-physical properties of nanofluids are provided in Appendix A.

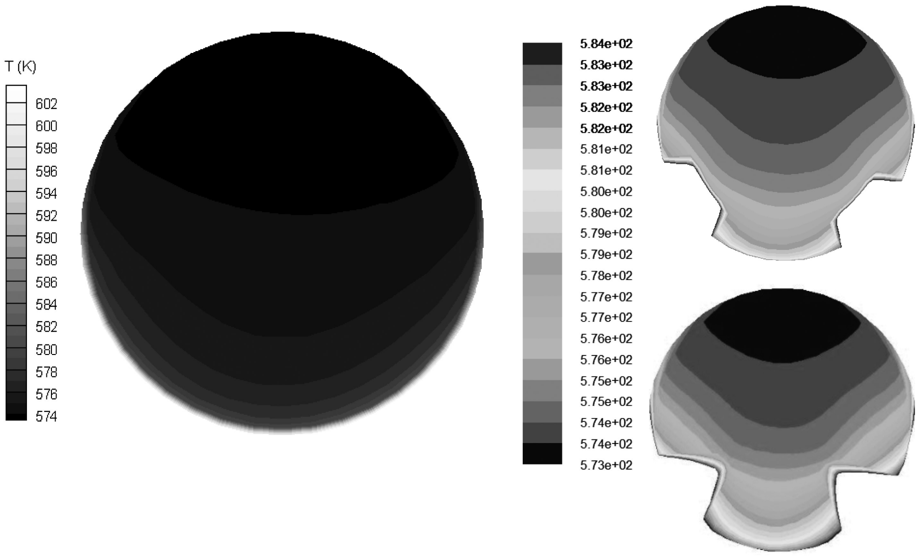
Fig. 10 represents the variation of the local Nusselt number of the four types of nanofluids used where DOWTHERM A was used as base fluid, with nanoparticle concentration 1% in volume and particle diameter of 13 nm.

The enhancement of heat transfer obtained by using nanofluids is due to the higher thermal conductivity of nanoparticles than normal fluids. The type of nanoparticle influences heat transfer; Fig. 10 shows the different results obtained with a number of nanofluids; the metallic nanoparticles improve heat transfer better than other nanoparticles.

Fig 11 indicates that using nanofluids as HTF in the absorber with fins can improve heat transfer greatly. It can also be seen that the influence of fins insert on heat transfer is much more significant than the presence of nanoparticles in the fluid. It should be noted that local Nusselt number and convective heat transfer coefficient decrease with axial distance.

In order, to estimate thermo-hydraulic performance of this compound technique, the criteria proposed by Bergles et al. [21] was used, considering constant pumping power as:

$$\eta = \frac{h}{h_{smooth}} \tag{13}$$




(a) Smooth absorber (b) Absorber with fins inserts

**Fig. 9** Variations of temperature distributions in the absorber

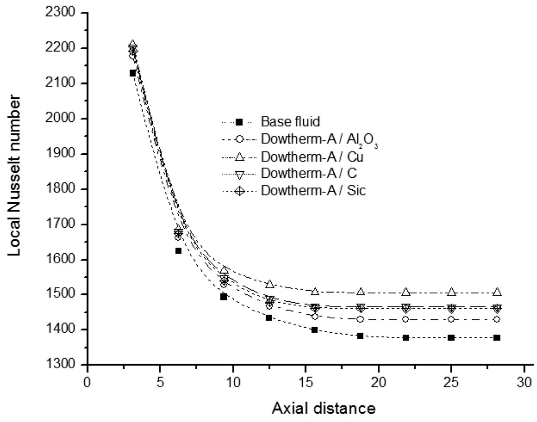



**Fig. 10** Variation of local Nusselt number in a smooth tube for Re= 257056 and φ=0.01.

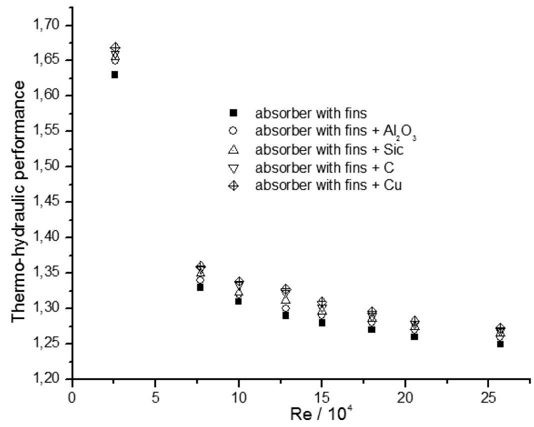



**Fig. 12** Thermo-hydraulic performance versus Reynolds number for $T_{in}$=573 K and φ=0.01

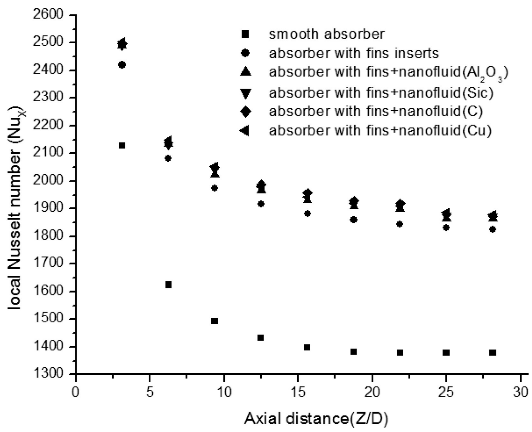



**Fig. 11** Effect of combining fins insert and nanofluids Re= 257056 and φ=0.01.

where: h is the convective heat transfer coefficient of the enhanced tube and, $h_{smooth}$ is the convective heat transfer coefficient of the smooth absorber.

It can be observed from Fig. 12 that, the efficiency decreases with increasing Reynolds number and the combination of both mechanisms provide better heat transfer while the enhancement factor varied from 1.3 to 1.68 when $2.57\times10^4 \le Re \le 2.57\times10^5$.

## Conclusion

In this paper, we have investigated the influence of heat transfer fluid properties and receiver geometries (with and without fins insert) of parabolic trough solar collector; on the whole the following conclusions can be made based on the results presented in this work:

- The Nusselt number for absorber with fins insert varied from 1.3 to 1.8 times in comparison with that of smooth tube.
- The friction factor for absorber with fins varied from 1.6 to 1.85 than plain tube.
- The geometric parameters of the fins have a remarkable effect in heat transfer improvement.
- Thermo-physical properties of nanofluid depend on physical characteristics and type of nanoparticle.
- The Nusselt number is responsive to type of nanoparticles used.
- The metallic nanoparticle enhances heat transfer greatly than other types.
- Higher enhancement results from combining the two mechanisms (fins and nanofluid).
- At similar condition, using nanofluid in absorber with fins insert offer higher heat transfer performance and higher thermo-hydraulic performance than smooth tube with base fluid.

## Appendix A

The thermo-physical properties of DOWTHERM A as a function of temperature [11]:

$$property = a + bT + cT^2 + dT^3 + eT^4 + fT^5$$

| property | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| Density (kg/m$^3$) | 1.493E+03 | -3.332E+00 | 1.248E-02 | -2.968E-05 | 3.444E-08 | -1.622E-11 |
| Specific heat (J/kg·K) | -2.364E+03 | 3.946E+01 | -1.703E-01 | 3.904E-04 | -4.422E-07 | 1.979E-10 |
| Conductivity (W/m·K) | 1.856E-01 | -1.600E-04 | 5.913E-12 | | | |
| Viscosity (Pa·s) | 5.135E+00 | -8.395E-02 | 5.971E-04 | -2.409E-06 | 6.029E-09 | -9.579E-12 |

Thermo-physical properties of nanofluid [22,23,24,25]

| | |
|---|---|
| Density | $\rho_{NF} = (1-\phi)\rho_F + \phi\rho_P$ |
| Specific heat | $c_{p,NF} = (1-\phi)c_{p,F} + \phi c_{p,P}$ |
| Viscosity | $\frac{\mu_{NF}}{M_F} = \frac{1}{(1-\phi)^{5/2}}$ |
| Thermal conductivity | $\lambda_{NF} = \lambda_F \frac{\Lambda_P + 2\Lambda_F - 2\phi(\Lambda_F - \Lambda_P)}{\Lambda_P + 2\Lambda_F + \phi(\Lambda_F - \Lambda_P)}$ |

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